3.a) Using the transfer function for 2.d) the following open-loop responses were recorded

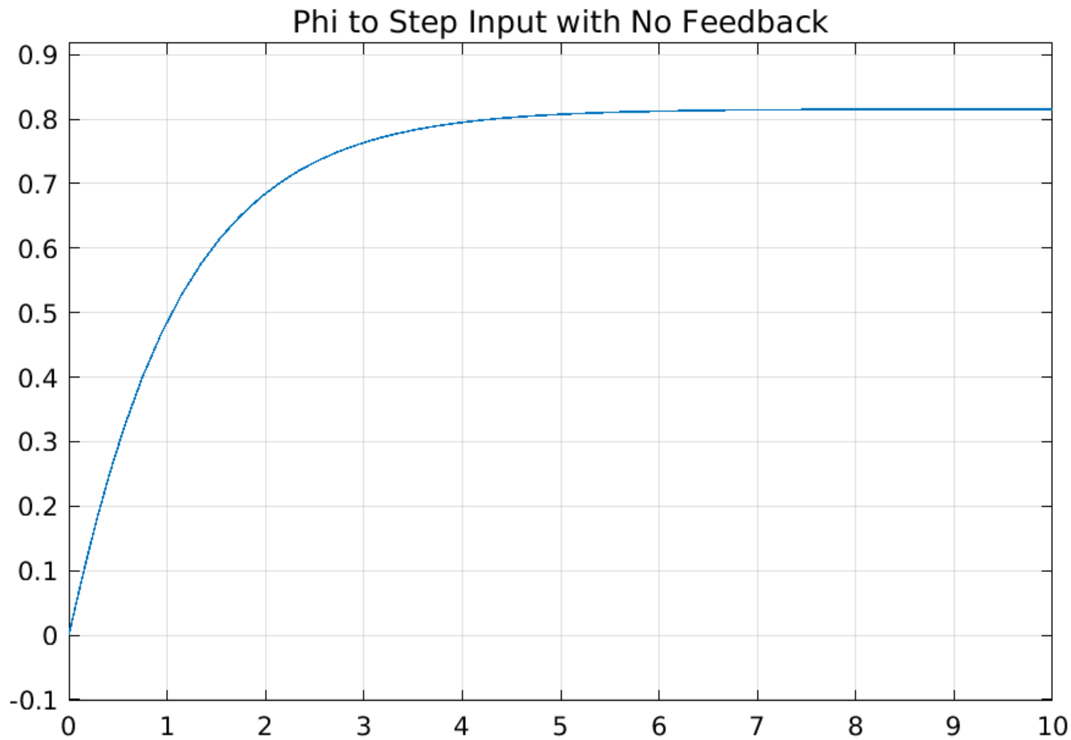

Figure 1: Open loop angle response to step velocity

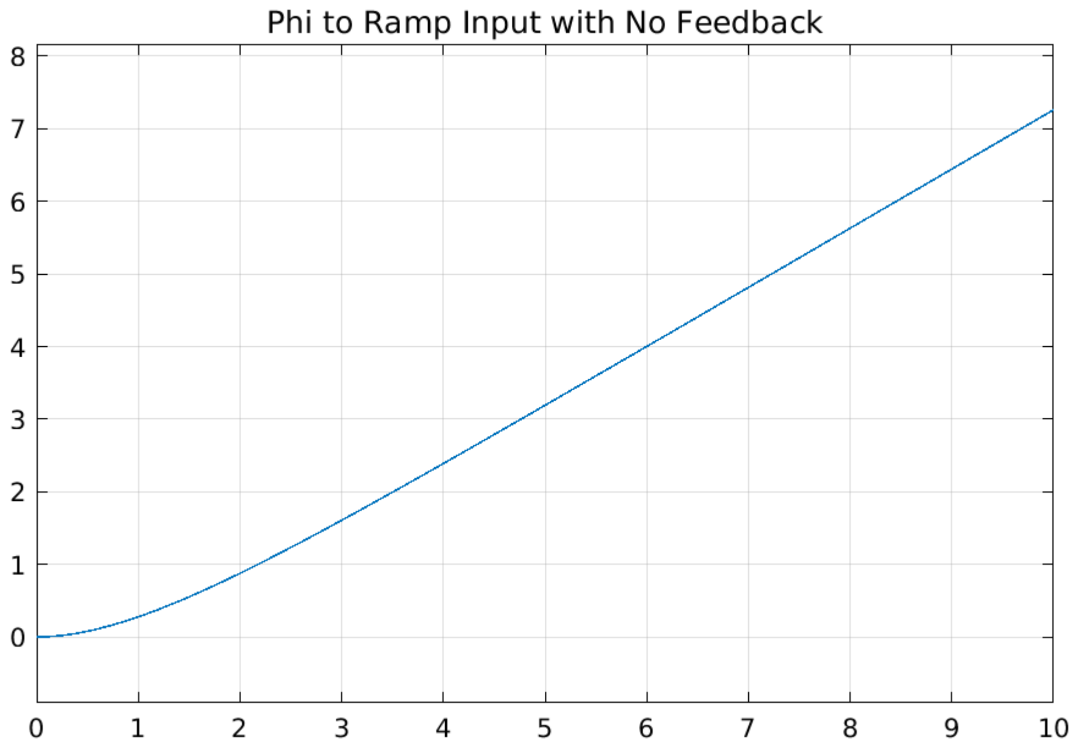

Figure 2: Open loop angle response to ramp velocity

3.b) Using the negative-unity feedback system for the transfer function from 2.d) the following responses were observed for unit step and ramp functions. Both responses seem to be roughly half the magnitude of their open-loop variants.

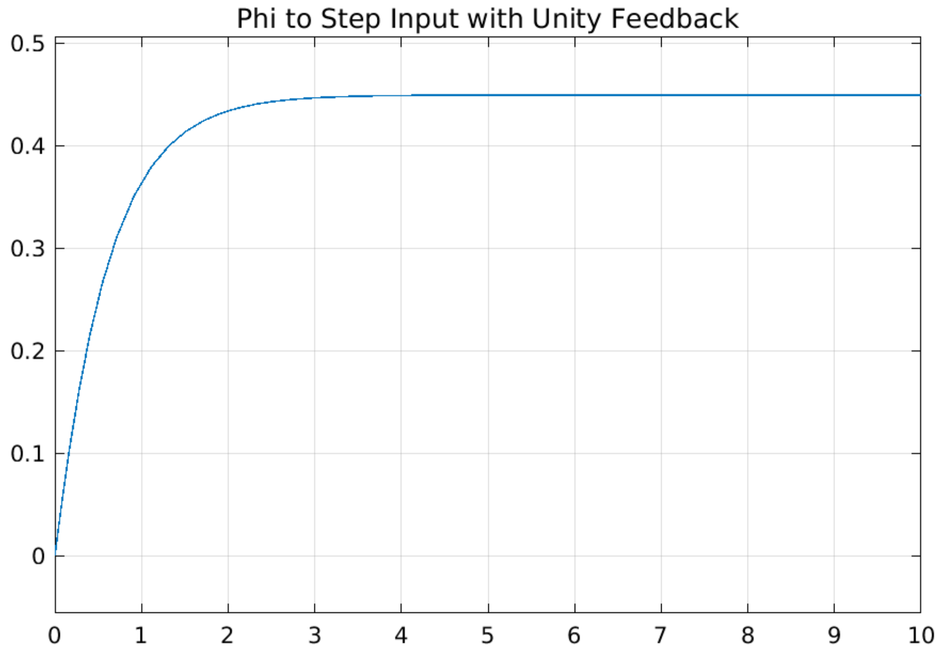

Figure 3: Angle response to step velocity with unity feedback

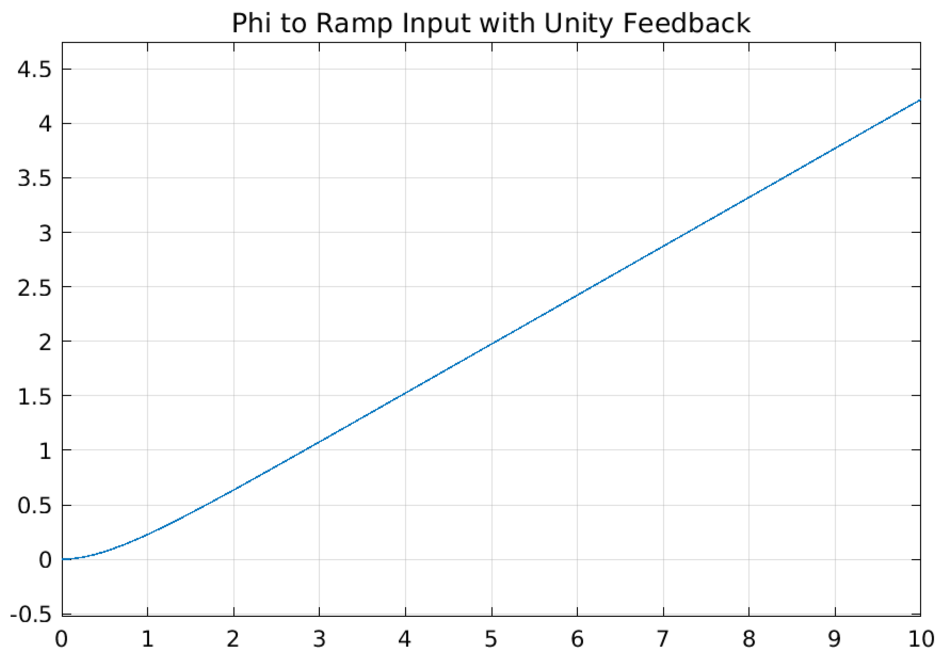

Figure 4: Angle response to ramp velocity with unity feedback

3.c) If I were to add a controller to this system in 3.b) I would add it between the output of the error junction and the system as shown below.

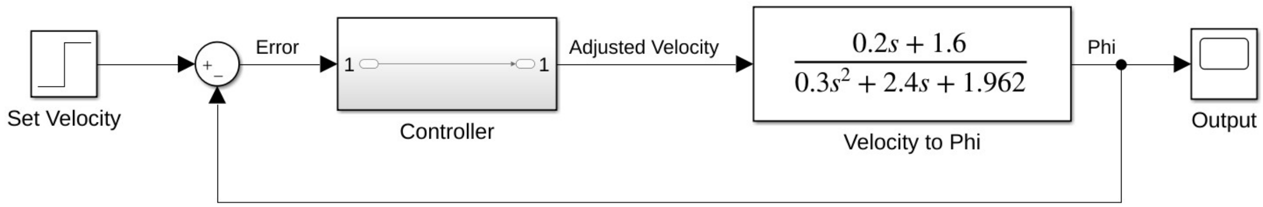

Figure 5: Proposed compensator location

The objective of this controller would be to adjust the system's response in some way by manipulating the error signal. This may be to minimize the impact of disturbances on the system or attain a better steady-state value. For this design project specifically we are to improve the transient response of angular displacement of the load (phi).

4.a) To apply the Routh-Hurwitz criteria to determine the stability of our system we need to know the forward transfer function (G(s)) and feedback transfer function (H(s)) of our system, derived below.

$$G(s)=\frac{\Phi}{V_t}=\frac{m_l s+b_l}{m_l L s^2+b_l L s+m_l g}=\frac{0.2 s+1.6}{0.2*1.5 s^2+1.6*1.5 s+0.2*9.81}=\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962}=\frac{\Phi}{V_t}$$

$$H(s)=1$$

These need to be combined to make a closed loop transfer function (T(s)) to be analyzed by using a Routh table.

$$T(s)=\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962}}{1+\left(\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962}\right)(1)}$$

$$T(s)=\frac{\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962}}{1+\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962}}\times\frac{0.3 s^2+2.4 s+1.962}{0.3 s^2+2.4 s+1.962}=\frac{0.2 s+1.6}{(0.3 s^2+2.4 s+1.962)+(0.2 s+1.6)}$$

$$T(s)=\frac{0.2 s+1.6}{0.3 s^2+2.4 s+1.962+0.2 s+1.6}=\frac{0.2 s+1.6}{0.3 s^2+2.6 s+3.562}$$

$$T(s)=\frac{0.2 s+1.6}{0.3 s^2+2.6 s+3.562}$$

This transfer function generates the following Routh table.

s^2	0.3	3.562
s^1	2.6	0
s^0	$\frac{-\begin{vmatrix}0.3 & 3.562\\ 2.6 & 0\end{vmatrix}}{2.6}=\frac{-(0-9.261)}{2.6}=3.562$	0

Going down the first column of the table there are no observed sign changes as all terms are positive. This means there are no poles in the right half of the imaginary plane, leaving both poles in the left half, thus the system is stable.

4.b) A controller was designed via root-locus by using Sisotool in MATLAB to meet the following two criteria:

- Percent overshoot is kept below 1%
- Settling time of less than 0.45 seconds

The first step in designing this controller was to transcribe the system's forward transfer function into MATLAB and then running the sisotool utility. This was achieved with the code below

```
clc             % Used to clear previous work
clear all
close all

s = tf('s'); % Used to define function

% Define transfer function and output to terminal
g = (0.2*s+1.6)/(0.3*s*s+2.4*s+1.962)

sisotool(g) % Open sisotool for this system
```

With sisotool opened it provided me with a root locus plot and step response plot for the initial system. These plots are shown below.

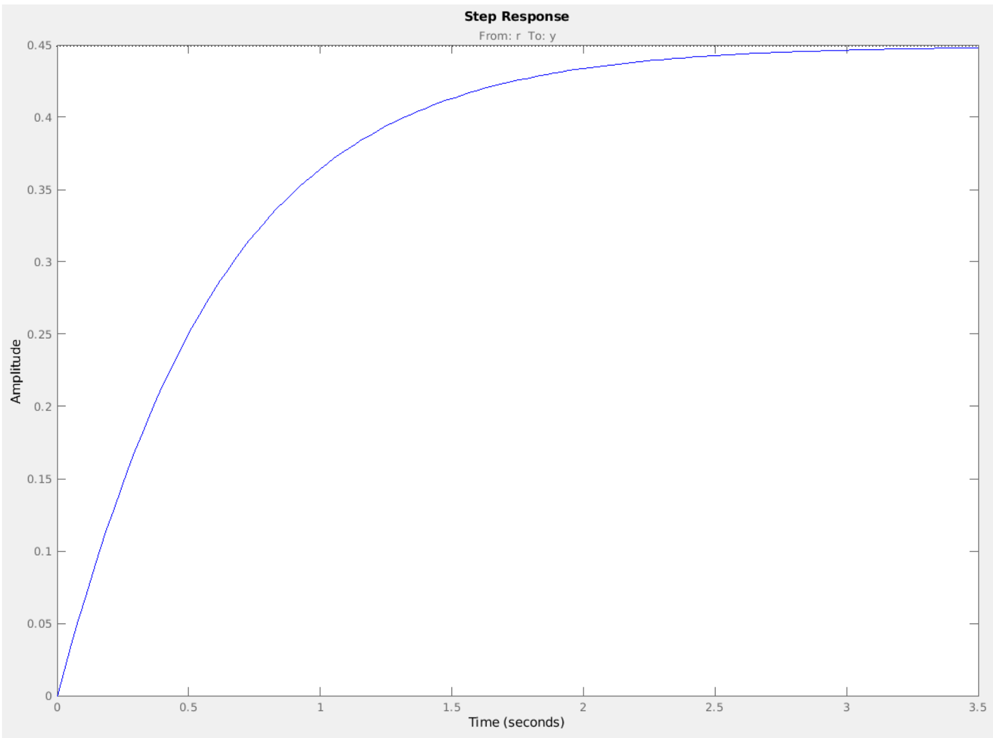

Figure 6: Step response of initial (uncompensated) system

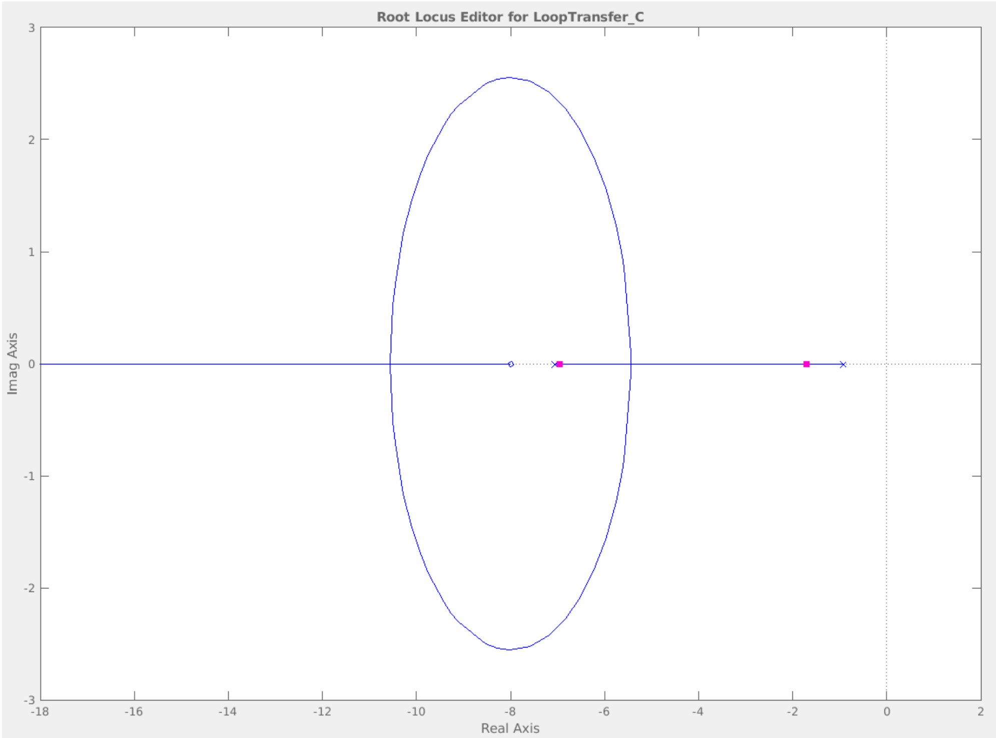

Figure 7: Root locus of initial (uncompensated) system in sisotool

After sisotool started, I added design requirements to match those specified to be met to the root locus graph. This added shaded regions that the dominant roots needed to be out of.

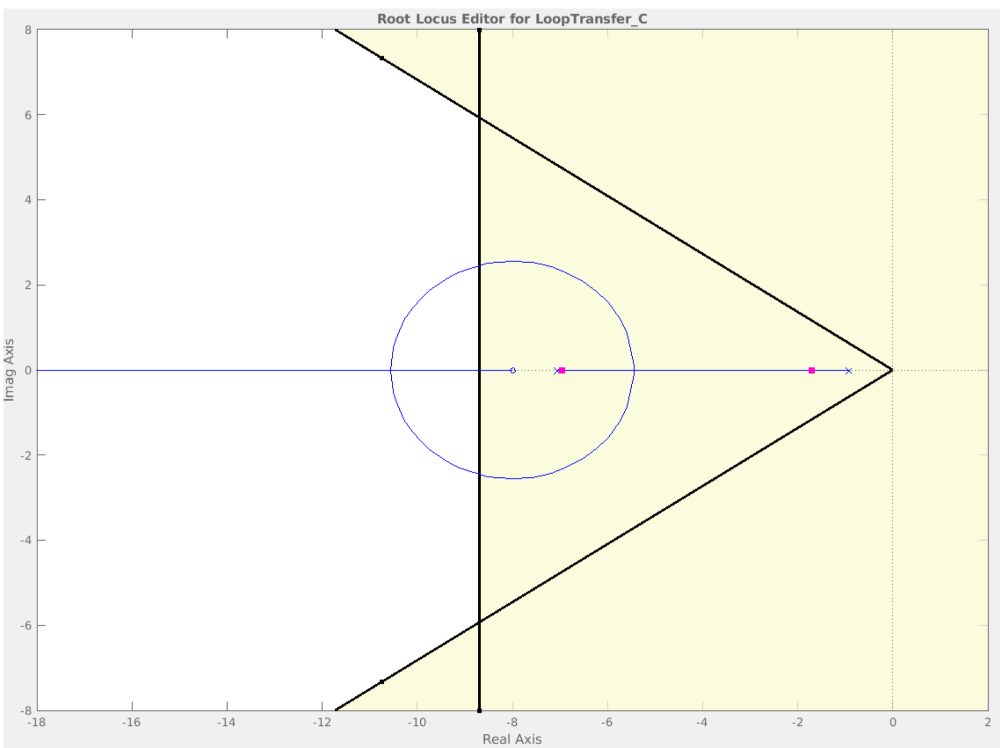

Figure 8: Root locus of initial (uncompensated) system with design requirement regions.

I then moved the dominant roots manually until they left the shaded regions. I settled on a K value of 20 which satisified both requirements. Below are the root locus plots and step response plots of the system with a gain (K) of 20 applied.

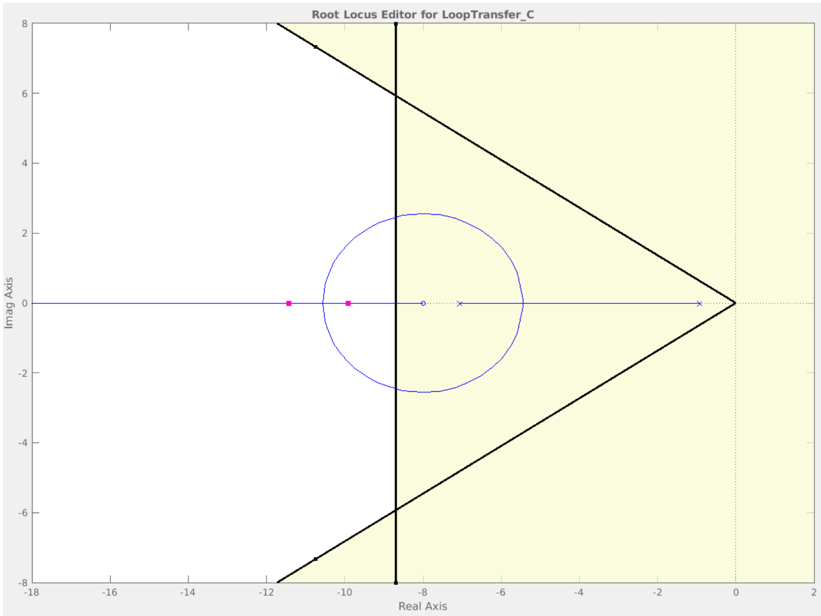

Figure 9: Root locus of system with a gain (K) of 20

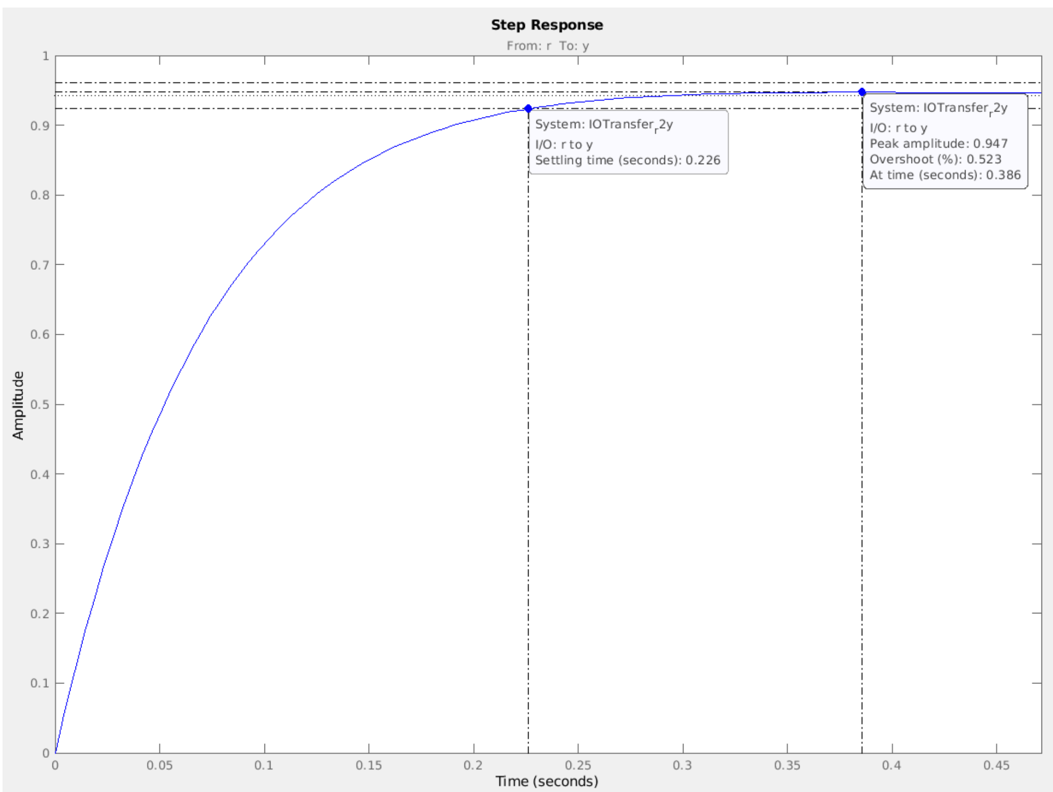

Figure 10: Step response plot of system with a gain (K) of 20

Given these results, **the compensator I propose for this system to meet the design requirements is a gain compensator set to a gain (K) of 20**. This design exceeds both requirements given our system configuration, summarized below.

Table 1: Design results compared to requirements

	Requirements	Design Characteristics
Settling Time (seconds)	< 0.45	0.226
Percent Overshoot (%)	< 1	0.523

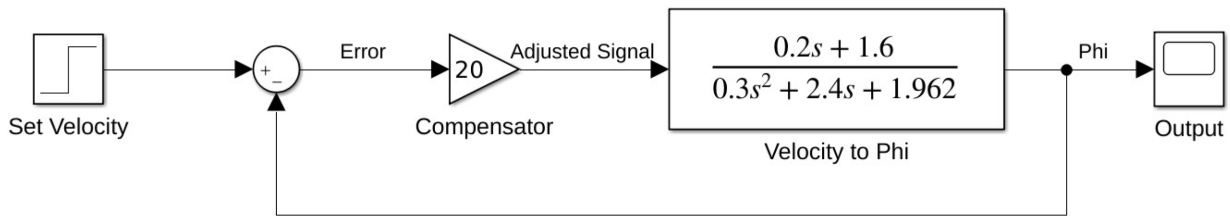

Figure 11: Proposed compensated system