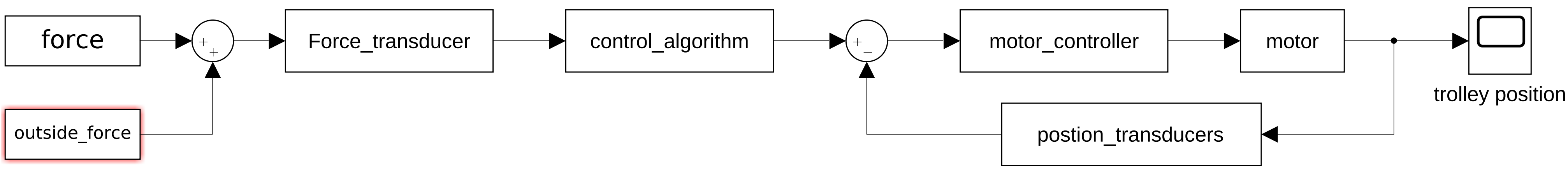
1.a)

Input	Outputs
x_T	ϕ
v_T	x_T
a_T	v_T
x_L	a_T
v_L	x_L
a_L	v_L
F	a_L

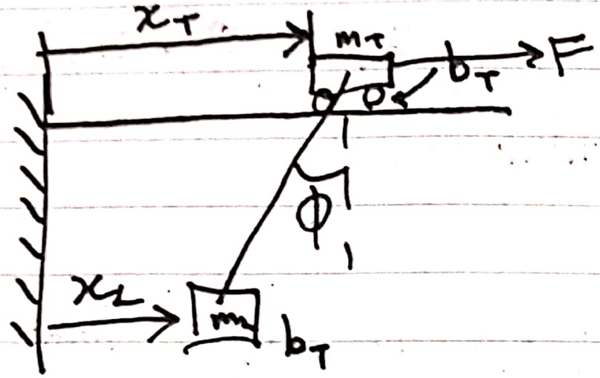
Any input can be paired with any output

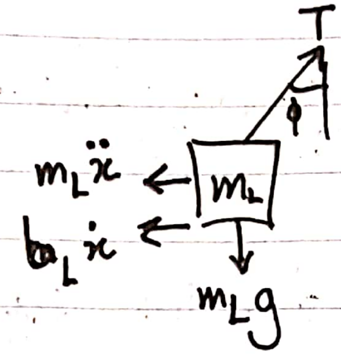
I would design a control system to ensure that the load is brought to the right location using the crane. To improve its efficiency of the crane we would design it to minimise ϕ, the angle the load line takes, this would require a closed loop system of us monitoring ϕ because it cannot be controlled directly, but as a result of the other controlled inputs.

1.B)


force
outside_force
Force_transducer
control_algorithm
motor_controller
motor
postion_transducers
trolley position

2.a)





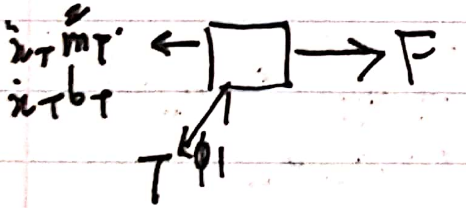
$$T\cos\phi = m_L g \quad \text{since} \quad \dot{y}_L = 0$$

$$\ddot{x}_L m_L + \dot{x}_L b_L = T\sin\phi$$

$$= m_L g \sin\phi / \cos\phi = m_L g \tan\phi$$

$$\ddot{x}_L m_L + \dot{x}_L b_L = m_L g \tan\phi$$

$$T = m_L g / \cos\phi = \sqrt{(m_L g)^2 + (m_L \ddot{x}_L + \dot{x}_L b_L)^2}$$



$$F = m_T \ddot{x}_T + \dot{x}_T b_T + T\sin\phi$$

or

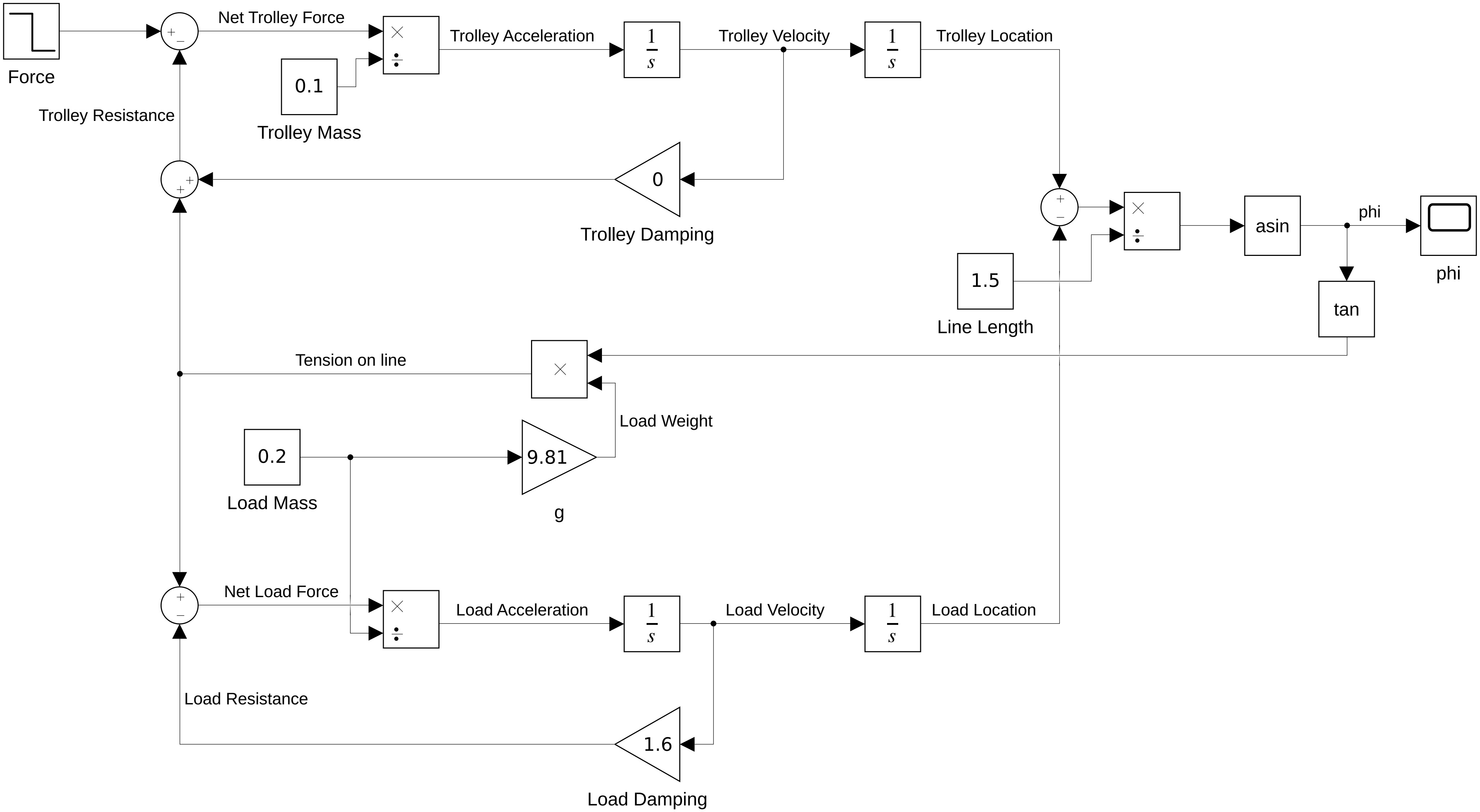
$$F = m_T \ddot{x}_T + \dot{x}_T b_T + m_L \ddot{x}_L + \dot{x}_L b_L$$

$$F = \ddot{x}_T m_T + \dot{x}_T b_T + m_L g \tan\phi$$

$$x_L = x_T - L\sin\phi$$

$$m_L g \phi = \ddot{x}_L m_L + \dot{x}_L b_L$$

$$F = x_T m_T + x_T b_T + m_L g \phi$$



Force
Net Trolley Force
Trolley Acceleration
Trolley Velocity
Trolley Location
0.1
Trolley Mass
Trolley Resistance
0
Trolley Damping
1.5
Line Length
asin
phi
tan
phi
Tension on line
Load Weight
0.2
Load Mass
9.81
g
Net Load Force
Load Acceleration
Load Velocity
Load Location
Load Resistance
1.6
Load Damping

2.c) Transfer function linking ϕ to x_T. Linearize the system assuming $\phi \approx 0$ prior to Laplace transform.

When $\phi \approx 0$: - $\sin(\phi) \doteq \phi$ $\quad \therefore \tan(\phi) = \phi$

$\cos(\phi) \doteq 1$

Linearized functions for the system:

$$F = \ddot{x}_T m_T + \dot{x}_T b_T + m_L g \phi \qquad (1)$$
$$m_L g \phi = \ddot{x}_L m_L + \dot{x}_L b_L \qquad (2)$$
$$x_L = x_T - L\phi \qquad (3)$$

Laplace transform

$$\mathcal{L}\{(1)\} = (4) \Rightarrow F = s^2 X_T m_T + s X_T b_T + m_L g \phi$$
$$\mathcal{L}\{(2)\} = (5) \Rightarrow m_L g \phi = s^2 X_L m_L + s X_L b_L$$
$$\mathcal{L}\{(3)\} = (6) \Rightarrow X_L = X_T - L\phi$$

Use (6) to express X_L as X_T and ϕ in (5):

$$m_L g \phi = s^2 X_L m_L + s X_L b_L \qquad X_L = X_T - L\phi$$
$$= X_L (s^2 m_L + s b_L)$$

$$m_L g \phi = (X_T - L\phi)(s^2 m_L + s b_L)$$
$$m_L g \phi + L(s^2 m_L + s b_L)\phi = X_T (s^2 m + s b_L)$$
$$(m_L g + s^2 L m_L + s L b_L)\phi = X_T (s^2 m + s b_L)$$

$$\boxed{\frac{\phi(s)}{X_T(s)} = \frac{s^2 m_L + s b_L}{s^2 L m_L + s L b_L + m_L g}}$$

Using the values required of my section $m_L = 0.2$ $m_T = 0.1$ $b_L = 1.6$ $L = 1.5$

$$\frac{\phi(s)}{X_T(s)} = \frac{s^2(0.2) + s(1.6)}{s^2(1.5)(0.2) + s(1.5)(1.6) + (0.2)(9.81)} = \frac{0.2s^2 + 1.6s}{0.3s^2 + 2.4s + \cancel{0.981}\ 1.962}$$

2.d) Find $\frac{\phi(s)}{V_T(s)}$ using same assuptions as .c)

$$F = \ddot{x}_T m_T + \dot{x}_T b_T + m_L g \phi = \dot{v}_T m_T + v_T b_T + m_L g \phi \quad , \quad v_T = \dot{x}_T \quad (1)$$

$$m_L g \phi = \ddot{x}_L m_L + \dot{x}_L b_L = \dot{v}_L m_L + v_L b_L \quad , \quad v_L = \dot{x}_L \quad (2)$$

$$L\phi = x_T - x_L \quad \cancel{= \int v_T - v_L \, dt}$$

$$\xrightarrow{d/dt} L\dot{\phi} = v_T - v_L \quad (3)$$

Laplace of all

$$\mathcal{L}\{(1)\} = (4) \Rightarrow F = sV_T m_T + V_T b_T + m_L g \Phi$$

$$\mathcal{L}\{(2)\} = (5) \Rightarrow m_L g \Phi = sV_L m_L + V_L b_L = V_L(sm_L + b_L)$$

$$\mathcal{L}\{(3)\} = (6) \Rightarrow sL\Phi = V_T - V_L$$

Rearrange (6) for V_L , $V_L = V_T - sL\Phi$ (7)

Use (7) in (5)

$$m_L g \Phi = V_L(sm_L + b_L) = (V_T - sL\Phi)(sm_L + b_L)$$

$$m_L g \Phi + sL(sm_L + b_L)\Phi = V_T(sm_L + b_L)$$

$$(m_L g + s^2 L m_L + sLb_L)\Phi = V_T(sm_L + b_L)$$

$$\boxed{\frac{\Phi(s)}{V_T(s)} = \frac{sm_L + b_L}{s^2 L m_L + sLb_L + m_L g}}$$

~~In my case this will result in~~

Using the values of my section $b_L = 1.6$ $L = 1.5$ $m_L = 0.2$ $m_T = 0.1$

$$\frac{\Phi(s)}{V_T(s)} = \frac{s(0.2) + 1.6}{s^2(1.5)(0.2) + s(1.5)(1.6) + (0.2)(9.81)} = \frac{0.2s + 1.6}{0.3s^2 + 2.4s + 1.962}$$